Localisation of partial discharges sources using acoustic transducers arrays

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This paper concerns the issue of the partial discharge (PD) sources location using acoustic emission transducer arrays technique and the high resolution direction of arrival estimation technology. In addition to the theoretical assumptions of both techniques, the simulation result, in which an uniform linear array (ULA) was used for registration of the acoustic emission signals generated by defects in power transformer insulation system were shown. To estimate the direction of arrival (DOA) of acoustic signals, that propagate from the discharge-generating defect to the transducer array installed on transformer tank, Multiple Signal Classification (MUSIC) algorithm was chosen. With the ability to locate multisource discharges (even at very low signal-to-noise ratio – SNR), the adopted solution has advantages over conventional techniques.

KEYWORDS: Partial Discharge, Signal Source Localisation, Transducer Array, Direction of Arrival (DOA) Estimation, MUSIC

1. Introduction

Defects in high-voltage insulation system, being a source of partial discharges (PD), are one of the major causes of failure of large power transformers. Issues relating to the detection, identification and localisation of PD sources are currently the subject of extensive research [1-6]. Their aim, among others, is development and improvement of the reliability of currently used diagnostic and monitoring techniques of power transformers based on the detection of PD phenomena, of which the main ones are: conventional electrical method (IEC 60270), electromagnetic methods (HF/NHF/UHF), acoustic emission method (AE) and dissolved gases analysis method (DGA). Authors’ research focuses on finding new theoretical and technological solutions that would greatly improve the accuracy of location of defects in high-voltage insulation system. One concept involves the use of sensor arrays techniques to estimate the direction of arrival (DOA) of the signal propagating from partial discharges. The origin of tackling this issue is the fact that the popular location techniques (standard and advanced auscultatory technique and triangulation technique), in a very difficult field conditions, i.e. the presence of multiple sources of PD, or recording noisy signals, does not allow to determine the expected accuracy of the defect in XYZ coordinates. Sensor array technology,
supported by the newest high resolution DOA estimation algorithms (MUSIC, Root-MUSIC, ESPRIT, MVDR etc.) is, at least theoretically, free from drawbacks and limitations of the aforementioned conventional techniques [7].

Further part of this article discusses the theoretical basis for the use of a linear AE transducers array and MUSIC algorithm for locating sources of partial discharges.

2. Linear transducers array – data model

Consider a Uniform Linear Array (ULA) consisting of $M$ identical, evenly spaced and located along a single line, measuring transducers. The distance between adjacent transducers is $\Delta$, and the distance from the defect (signal source) to the first receiver of transducers array (looking from the right) is $d_d$.

The signal generated by the source is defined as [8]:

$$s_i(t) = \alpha_i(t) \cos(2\pi f_c t + \beta_i(t)), \quad 1 \leq i \leq d$$  \hspace{1cm} (1)

where: $\alpha_i(t)$ – signals amplitude, $f_c$ – carrier frequency, $\beta_i(t)$ – phases, $d$ – number of sources.

Let assume that these signals are narrowband. This means that the amplitude $\alpha_i(t)$ and phase $\beta_i(t)$ changes slowly with respect to $\tau$, which is the wave propagation time between successive elements of the array (Fig. 1). Therefore:

$$\alpha_i(t - \tau) \approx \alpha_i(t), \hspace{1cm} \beta_i(t - \tau) \approx \beta_i(t).$$  \hspace{1cm} (2)

Slowly varying amplitudes $\alpha_i(t)$ and phases $\beta_i(t)$ ensure that as a result of the Fourier transform of equation (1), the majority of the frequency components in close proximity to the dominant frequency $f_c$ will be obtained. Equation (1) can also be represented as a complex envelope, or in the form of so-called Phasor: $s_i^{env}(t) = \alpha_i(t)e^{j\beta_i(t)}$, such as $s_i^{env}(t) = \text{Re}(s_i^{env}(t)e^{j2\pi f_c t})$.

Fig. 1. Scenario under consideration in the article
Now let assume that a plane wave generated by the source \( i \) reaches the array with velocity \( v \) and angle \( \theta_i \) (Fig. 2).

![Diagram of data model for DOA estimation of \( d \) sources with a linear array of the \( M \) element](image)

Fig. 2. Data model for DOA estimation of \( d \) sources with a linear array of the \( M \) element

The signal coming through distance \( d_i \) arrives first at the transducer closest to the source after time \( \tau_d = \frac{d_i}{v} \). It can be written as:

\[
s_{i1}(t) = s_i'(t - \tau_d) = \alpha_i(t - \tau_d)\cos[2\pi f_c(t - \tau_d) + \beta_i(t - \tau_d)] = \text{Re}\left\{ s_i(t) \right\} = \text{Re}\left\{ \alpha_i(t - \tau_d)e^{j2\pi f_c(t - \tau_d) + \beta_i(t - \tau_d)} \right\}.
\]  

(3)

Because all the receiving transducers of the array are located along a single line, the signal received by the \( m \)-th transmitter travels, as compared to the signal reaching the extreme right (first) converter, a further length, which can be determined from the following relationship:

\[
\Delta_{mi} = (m-1)\Delta \sin \theta_i, \quad m = 1, 2, \ldots, M.
\]  

(4)

Let assume that the signal reaches the \( m \)-th transmitter with delay \( \tau_{mi} \):

\[
\tau_{mi} = \frac{\Delta_{mi}}{v} = (m-1)\frac{\Delta \sin \theta_i}{v}.
\]  

(5)

Therefore, the signal registered by the \( m \)-th sensor can be defined as a delayed version of the signal \( s_{i1}(t) \) (recorded by the first transducer rightmost) with an additional delay \( \tau_{mi} \):

\[
s_{im}(t) = s_{i1}(t - \tau_{mi}) = s_i'(t - \tau_d - \tau_{mi}) = \alpha_i(t - \tau_d - \tau_{mi})\cos[2\pi f_c(t - \tau_d - \tau_{mi}) + \beta_i(t - \tau_d - \tau_{mi})] = \text{Re}\left\{ s_i(t)e^{j(\pi f_c t - \pi f_c \tau_d - \pi f_c \tau_{mi} + \beta_i t - \beta_i \tau_d - (m-1)\beta_i)} \right\}.
\]  

(6)
where \( \mu_i = \frac{2\pi f_c}{\lambda} \Delta \sin \theta_i = \frac{2\pi}{\lambda} \Delta \sin \theta_i \) — spatial frequency associated with \( i \)-th signal source, generating signal at incidence angle \( \theta_i \); \( \lambda = \frac{v}{f_c} \) — wavelength corresponding to the carrier frequency \( f_c \).

In determining the equation (6) the approximation (2) was taken into account. In the complex form, the above received signals correspond to:

\[
s_{im}(t) \approx \alpha_i (t - \tau_d) e^{j2\pi f_c (t - \tau_d) + j(M-1)\mu_i} = s_i(t) e^{j(M-1)\mu_i}. \tag{7}
\]

Equation (7) shows that the signal \( s_{im}(t) \), registered by the \( m \)-th transmitter, which was generated by the \( i \)-th source, is identical to the signal \( s_i(t) \) recorded by the first (extreme right) transducer, but with an additional phase shift factor \( e^{j(M-1)\mu_i} \). This factor is dependent only on spatial frequency \( \mu_i \) and the position of the array element relative to the first element. For each incidence angle \( \theta_i \), there is a corresponding spatial frequency \( \mu_i \). Therefore, the primary goal of estimation of the direction of arrival of the signal, is to extract spatial frequency \( \mu_i \) from the signals received by the transducers array. It is important to fulfil the condition of minimum distance between the transducers \( \Delta \), which should be less than or equal to half the wavelength \( \lambda \).

Now consider the situation where all the signals \( s_i(t) \), generated by \( d \)-sources, and the noise \( n_m(t) \) received by the \( m \)-th transmitter at the time \( t \), can be represented by the following equation:

\[
x_m(t) = \sum_{i=1}^{d} s_i(t) + n_m(t) = \sum_{i=1}^{d} s_i(t) e^{j(M-1)\mu_i} + n_m(t) = s_i(t) \sum_{j=1}^{d} e^{j(M-1)\mu_i} + n_m(t), \tag{8}
\]

In a matrix form equation (8) can be written as:

\[
x(t) = \begin{bmatrix} a(\mu_1) & a(\mu_2) & \ldots & a(\mu_d) \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_d(t) \end{bmatrix} + n(t) = As(t) + n(t), \tag{9}
\]

where: \( x(t) = [x_1(t) \ x_2(t) \ \ldots \ x_M(t)]^T \) — data column vector received by the array, \( s(t) = [s_1(t) \ s_2(t) \ \ldots \ s_M(t)]^T \) — signal column vector generated by the sources, \( n(t) = [n_1(t) \ n_2(t) \ \ldots \ n_M(t)]^T \) — zero-mean Gaussian noise.

The array steering column vector is defined as (spatial frequencies \( \mu_i \) are unknown):

\[
a(\mu_i) = \begin{bmatrix} 1 & e^{j\mu_1} & e^{j2\mu_1} & \ldots & e^{j(M-1)\mu_1} \end{bmatrix}^T, \tag{10}
\]

written in the form of a matrix (of size \( M \times d \)):
3. Direction of arrival estimation using multiple signal classification (MUSIC) algorithm

MUSIC (Multiple Signal Classification) algorithm is one of the most widely used techniques of high resolution direction of arrival estimation. It belongs to the group of the subspace methods and the operation of the algorithm can be briefly presented in the following steps:

- **Step 1:** Register input signals \( x(t_n), n = 1, 2, \ldots, N \) and make the estimation of the covariance matrix:

  \[
  R_{xx} \approx \hat{R}_{xx} = \frac{1}{N} \sum_{n=1}^{N} x(t_n)x^H(t_n). \tag{12}
  \]

- **Step 2:** Perform the decomposition of the covariance matrix relative of the eigenvalues

  \[
  \hat{R}_{xx}V = VA, \tag{13}
  \]

  where \( \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_M) \), \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M \) are the eigenvalues and \( V \) contains all the eigenvectors \( \hat{R}_{xx} \).

- **Step 3:** Estimate the multiplicity \( k \) of the smallest eigenvalue \( \lambda_{\text{min}} \) and then the number of signals \( d \) from as:

  \[
  d = M - k. \tag{14}
  \]

- **Step 4:** Determine the MUSIC frequency spectrum:

  \[
  P(\theta) = P_{\text{MUSIC}}(\theta) = \frac{1}{a^H(\theta)\hat{V}_n^H\hat{V}_n a(\theta)}, \tag{15}
  \]

  where \( \hat{V}_n = [q_{d+1}, \ldots, q_M] \) with \( q_l = d+1, d+2, \ldots, M \) being the eigenvectors corresponding the smallest eigenvalue \( \lambda_{\text{min}} \).

- **Step 5:** Find \( d \) pick values in \( P_{\text{MUSIC}}(\theta) \) frequency spectrum, which correspond the DOAs.

4. Localisation of PD sources in power transformer tank – simulation results

Partial discharges, depending on the type of the defect and the insulating system in which they occur, emit AE waves in wide range of frequencies (30
kHz – 600 kHz). In performed simulations, an AE signal was modeled as a combination of sine and exponential functions:

\[
 f(t) = \begin{cases} 
 0.5e^{(-2 \cdot 10^9 (t - t_0))} \sin(2\pi ft) & \text{if } 0 \leq t \leq t_t \\
 0.5e^{(-2 \cdot 10^9 (t - t_0))} \sin(2\pi ft) & \text{if } t_0 < t < t_1
\end{cases}
\]

where: \( t_0 = 0.001 \text{s} \) and \( t_1 = 0.004 \text{s} \) (16)

with the frequency \( f=110 \text{ kHz} \). The value of \( f \) was dictated by the fact, that it is the dominant frequency of surface discharges pulses (registered with PAC WD transducer) which pose the greatest threat to the paper-oil insulation system of the transformer [6]. In order to reflect, as precisely as possible, difficult measuring conditions prevailing during actual diagnostic tests (high levels of noise and broadband interference), Gaussian white noise was added to the clean harmonic signals. Such modeled signals were characterized by a very low Signal-to-Noise Ratio (SNR) (Fig. 3).

For receiving signals a mathematical model of Uniform Linear Array consisting of four sensors, with parameters corresponding to the popular broadband piezoelectric transducers type of PAC WD (frequency response: 100-1000 kHz; the resonant frequencies: 125 kHz, 200 kHz, 280 kHz, 420 kHz, 530 kHz, directivity: ±1.5 dB) was used [6]. To fulfill the condition of minimum distance between the elements of the array, which should be less than, or equal to half of the wavelength \( \lambda \), it is assumed that they are spaced at intervals of \( \Delta = 5 \text{ mm} \).

It should be emphasized that with the ULA's it is possible to determine only the azimuth angle \( (\theta) \). In order to estimate the direction of arrival in three dimensions, it is necessary to have knowledge of the elevation angle. This is possible only through the use of at least two-dimensional array (e.g. circular or rectangular).

Therefore, the conducted simulations assumed simplification that the source of the acoustic emission signal generated by the PD is on the same level as the transducers array. As a result of this assumption the Z coordinate, which is the coordinate of the third dimension, was omitted (Fig. 4).
As an object of study, a power transformer tank was modeled, with length \( x = 6 \text{ m} \) and width \( y = 3 \text{ m} \). It was assumed, that the point with coordinates \([0,0]\) is located in the lower left corner of the tank model. To determine the \( XY \) coordinates of the defect, sensor array must be placed in, at least, two different places on the tank (as a result of that, two different DOAs are obtained). Intersection point of two lines, led at designated angles from the center of the arrays, will determine the source location.

In performed simulations, transducers arrays were placed on the \( x \)-axis (front wall of the transformer tank) at a distance of two and four meters from the origin \([0,0]\) (Fig. 4).

Scenario adopted in the simulation consisted of two cases. In first case (marked as \( A \)) the \( AE \) signal was generated by the source with coordinates \( x = 5.5 \text{ m} \) and \( y = 2.5 \text{ m} \), in the second case (\( B \)) the signal source was located in \( x = 2.5 \text{ m} \) and \( y = 1 \text{ m} \) (Fig. 5).

![Fig. 4. Schematic model of power transformer tank (top view) with marked locations of the transducers arrays and the angle of signal arrival determining method](image)

![Fig. 5. Simulation of PD sources localisation using linear AE transducers array: a) location of simulated defects (case \( A \) and \( B \)), b) example result for clean signal (\( SNR \rightarrow \infty \)), c) example result for highly noisy signal (\( SNR = -14 \text{ dB} \))](image)
The results of the simulation showed that by using a high-resolution frequency spectrum estimation algorithm (MUSIC), the error of the localised coordinates of the partial discharge source is negligibly small. For highly noisy signals the average error of the localisation was 5.5 cm along the OX axis and 5.2 cm along the OY axis.

![Diagram](image)

**Fig. 6. PD source localisation errors (in [cm]) drawn in the Cartesian coordinate system (the [0,0] point stands for the real defect location)**

### 5. Conclusion

The article presents the results of research concerning the determination of the applicability of the acoustic emission transducers array and high-resolution algorithm (MUSIC) for estimating the direction of arrival to the localisation of partial discharges sources in power transformers. The results of simulations suggest that new method proposed by the authors may be a preferred alternative to the conventional triangulation technique (especially when the recorded signals are highly noisy, or for the multisource discharges). The next stage of the planned research will include the design and construction of a two-dimensional transducers array and further laboratory tests on a real model of the transformer tank.

### References


