Background noise distribution for noise object tracking in Track-Before-Detect systems using minimal chi-square statistic

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Tracking of the noise signal in noise measurements needs special techniques. The difference between object and background noise is defined, using the noise distribution. The proposed technique is based on the model of the background noise. The window based approach is used for input signal preprocessing. The comparison of two distributions – empirical and observed is used. The global distribution is obtained using all measurements and the observed distribution is computed in the window area only. Minimal chi-square statistic is used as comparison criteria and results are processed by Spatio-Temporal Track-Before-Detect algorithm for tracking of the dynamic of the object and improved signal denoising. A few examples are shown for different objects that show possibilities of the proposed solution. Mean value suppression is possible using comparison of both distributions, what is important in application where the background estimation is not ideal.

1. Introduction

Tracking systems are used for estimation of the variable parameters of the object [2-4]. The filtering ability of the tracking algorithms reduces the noise influences. Threshold based techniques are not feasible for noised measurements, where SNR is near or below unity values. In most cases tracked signal of the object are stable (constant value) or quasi-stable (constant value with additive variable signals). Demodulation techniques are feasible for variable signals, also. Special case of the signal is the case where the signal is noise only. The noise values are different for every measurement, so appropriate preprocessing of the measurements is desired. All measurements are the noise values only, and the difference between signal and distortions are related to the local statistical properties of the measurements, defined in time or spatial domain.

Such technique is considered in [16], where the standard deviation is calculated using the moving window approach. Such technique allows separation of two signals: background and object using local standard deviation. Results of preprocessing are inputs of the tracking algorithm for further detection improving. Standard deviation is simple to calculate, but two noise sets from two different distributions may have equal standard deviations. Such signals cannot be distinguished, and more robust technique is necessary.

The next problem is that the obtained preprocessed values are low-valued. It is important problem for noise signals, especially. The limited number of the samples
is important for the high resolution (e.g. position). It is also problem with estimation of stable values of estimators. The reduced number of values influences on the noise in the mean, standard deviation and other estimators. Additional filtering, due to multiple measurements is necessary. Track-Before-Detect (TBD) algorithms [4, 5, 19, 20] are especially important for noise signals [16]. The separation of both signals is possible, even if the signal (noise) is hidden in the background noise.

Noise objects needs noise preprocessing (estimation of the noise values) and TBD processing. Additional technique allows comparison of empirical and observed distribution without consideration of the mean value. The mean value could be a part of the signal (noise and additive constant value), the disturbing effects of measurement system or countermeasure. The constant or smooth variable changes are considered as signal (in this case noise only) disturbance.

![Proposed system for noises separation using minimal chi-square statistic and TBD algorithm](image)

Fig. 1. Proposed system for noises separation using minimal chi-square statistic and TBD algorithm

2. Minimal chi-square statistic

This technique is based on the application of the chi-square statistic for comparison of two distributions [1, 6]. The first distribution is empirical and is calculated using overall measurements. Obtained values are used as a basis of the distribution sampling. The distribution is divided on regions, and a set of subareas are obtained. All of them have similar field area values. It is not necessary to divide distribution on equal subarea parts. The boundaries between subareas are also established. An example of subareas of the standard normal distribution is shown in Fig. 2.

Another dividing technique is based on the fixed width of subareas (fixed regions). Such technique allows application of the histogram measurement technique, what is computationally important. Calculation of the histogram for the fixed and equally spaced regions is based on counting of the events. Assignment of the sample value to the appropriate regions is direct and allows incrementation of the particular counter, corresponding to this region. Calculation of the empirical discreet distribution is more complicated, due to necessary fitting of regions. It is quite simple for the typical distribution like normal one, but the a priori knowledge about empirical distribution is necessary.

Equally spaced regions could be used for calculation of histogram for any distribution. Such approach needs carefully implementation of the histogram calculation, due to sampling limitations. Dense regions are desired for sharp distributions, but some regions may be not filled by the empirical data. Large data
set for the estimation of empirical distribution is necessary. Such requirements could be fulfilled, if the background noise is modelled, not object. The number of samples related to the object is usually smaller in comparison to the background. Such approach assumes that empirical distribution is calculated using the same stochastic process. Spatial or temporal changes of distribution disturb the detection performance.

The local measurements are related to the moving window approach. Every window position selects the fixed number of values in the window range. The selected values are assigned to the corresponding subarea and counted. It allows estimation of the second distribution (observation) - \(O\).

Both distributions are compared using the following formula:

\[
\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}
\]  

(1)
The empirical distribution $E$ related to the background noise must be positive. Zero-valued cases are possible for the empirical distribution that is calculated using data empirical data. Dense sampling increases the possibility of such cases. Positive only cases should be compared only, or other criteria should be used instead of the chi-square.

$$\chi^2 = \sum_{E_i>0} \frac{(O_i - E_i)^2}{E_i}$$  \hspace{1cm} (2)

The chi-square value is non-negative one. Testing of this value is important for the detection algorithm. Conventional tracking systems are based on the detection and tracking approach, but low SNR case are not possible in processing. Alternative approach (TBD) assumes tracking of all possible trajectories before the detection. It allows noise reduction using multiple measurements. The chi-square tests are not important for TBD systems and the values are used directly as input data $X$ for TBD algorithm (3c).

The chi-square and window approach may be applied for the detection of signal (Fig. 4). In the following example, the object’s signal is Gaussian noise (standard deviation = 5), and the background noise is also Gaussian noise (standard deviation = 2.5). Mean value related to the object is assumed equal to the 1. There are 7 of regions in distributions with boundaries $-\text{Inf}$, $-2.5$, $-1.5$, $-0.5$, $0.5$, $1.5$, $2.5$, $+\text{Inf}$. Empirical distribution obtained for the background and object is shown in Fig. 5 (All samples are used for the estimation of background noise properties (mean and standard deviation)). The length of the window analysis is 20 and the object size (in time or spatial domain) is 20, also.

The suppression of the unknown mean value and the comparison of the noises (background and related to the object) is based on the window technique for chi-square statistic:

$$\chi(j)^2 = \sum_{E_i>0} \frac{(O_{ij} - E_i)^2}{E_i}$$  \hspace{1cm} (3)

The output value is the minimal chi-square value (4), obtained for possible offsets of observed distribution. Some boundary limits are necessary. The following formula is used:

$$\chi_{\text{min}}^2 = \min \chi(j)^2 = \sum_{E_i>0} \frac{(O_{ij} - E_i)^2}{E_i}$$  \hspace{1cm} (4)
Fig. 4. Preprocessing of the input measurements using minimal chi-square statistic with fixed regions

Such technique allows preprocessing of the input measurements, and the conventional threshold technique is sufficient for the detection of local changes. This example shows possibility of the detection but there are many high-value chi-square peaks, which are also possible positions. This effect of the short sample size (window size) applied for the estimation of chi-square value. Tracking of the peaks over many measurements using TBD algorithm is necessary.

3. Spatio-temporal Track-Before_Detect algorithm

There are numerous TBD algorithms and the Spatio-Temporal TBD algorithm may use recursive processing for reduced number of computation. This algorithm processes all possible trajectories. Tracking ability allows processing multiple measurements for emphasis of the weak signal of the object, even if this signal changes values and position.

Start

\[ P(k = 0, s) = 0 \]  // initialization \hfill (5a)

For \( k \geq 1 \)

\[ P^-(k, s) = \int_S q_k(s \mid s_{k-1}) P(k - 1, s_{k-1}) ds_{k-1} \]  // motion update \hfill (5b)

\[ P(k, s) = \alpha P^-(k, s) + (1 - \alpha) X(k, s) \]  // information update \hfill (5c)

EndFor

Stop

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where: \( k \) – iteration number, \( s \) – particular space, \( X \) – input data, \( P^- \) – predicted TBD output, \( P \) – TBD output, \( \alpha \) – weight (smoothing coefficient), range: 0-1.

The ST-TBD algorithm is a kind of the exponential filter and the information update formula is responsible for this behaviour. Chi-square and minimal chi-square values have lower boundary fixed and the minimal value is 0. There is not upper boundary, what is important disadvantage.

The large, non limited values influents on the preservations of the trajectory and it is important disadvantage if the manoeuvre occurs. Switching between trajectories should be possible. Suppression techniques [6, 8, 12, 15, 17, 18] should be used for range reduction.

4. Gaussian signal and additive Gaussian noise

In this example two signals are additive. The standard deviation of the Gaussian signal is 1, and the standard deviation of the Gaussian noise is 1. The signal is not moving, so single motion vector is depicted, only. Two noises are additive so standard deviation is increased in common area (central part of the measurement). Smoothing coefficient is \( \alpha = 0.95 \).

The window has 20 sample sizes. Predicted values are depicted in Fig. 5. The white strip corresponds to the tracked object.

![Fig. 5. Example results (the most important subspace) for Gaussian signal and additive Gaussian noise of background](image-url)

In this example the minimal chi-square statistic allows detection of the difference between overall standard deviation and local standard deviation, related to the signal.
5. Gaussian signal and additive Gaussian noise with different mean values

The case, where the signal is Gaussian noise with additional constant value, in this example (Fig. 6) is shown. The TBD systems have ability of the detection of this constant value if the chi-square statistic is applied [6], but it is interesting how this signal is processed for the minimal chi-square statistic.

The most important difference between both statistics is the suppression of the constant level by the minimal chi-square. Two kinds of effect occur. First is related to the suppression of the mean value of the signal according to formula (4). Second is related to the number of regions that influent on the discreet distribution.

Reduced number of regions (bins) changes distribution if the constant level is high. The distribution of the signal is changed due to one-side saturation. The positive constant level of signal is influenced by the right side saturation. High constant level may change histogram to the single region (bin).

In Fig. 6 is shown result for the boundaries: −Inf, -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, +Inf. In Fig. 7 is shown result for the larger number of boundaries where the saturation does not occur.

Saturation influences on changes of the distribution and may improve detection of the signal with constant level (Fig. 7 – additive value +2.0), but is not related to the noise of signal directly. Constant level signal without noise is also detectable, what is show in Fig. 8.
Fig. 7. Example results (the most important subspace) for Gaussian noise signal with constant value and additive Gaussian noise of background – without saturation

Fig. 8. Example results (the most important subspace) for constant value signal and additive Gaussian noise of background – saturation effect
6. Gaussian mixtures and additive Gaussian noise (signal)

In this example (Fig. 10) the minimal chi-square statistic allows detection of the difference between distributions. The Gaussian mixtures are used for the description of more complex distributions by the composition of multiple Gaussian distributions with different means and standard deviations (Fig. 9). The fixed region approach is applied to the complex distribution that is modeled as two Gaussian distributions with standard deviation 1 and mean values -2 and +2. The signal is the Gaussian distribution with standard deviation 1 and mean value 0. There are a following boundaries: -Inf, -6.5, -5.5, -4.5, -3.5, -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5 and +Inf.

Fig. 9. Gaussian mixture (background noise) and Gaussian signal

Fig. 10. Example results for Gaussian mixture (background noise) and Gaussian signal

Proposed technique allows tracking the non-Gaussian signals also. Dispersion of the trajectory is related to the window size.
8. Conclusion and further works

The proposed technique for tracking of the noise objects is important technique that is interesting for the analysis of the signals. Noises are typical for measurements, and they occur due to measurement problems, external influences or are related to the considered process. Diagnosis of process, estimation of the state of devices using such technique is possible. The Chi-square preprocessing for TBD algorithm is rather simple [6], from the computation point-of-view in comparison to the TBD itself [9-11, 13, 14]. Minimal chi-square preprocessing allows suppression of constant value signals (only noises are compared).

Real-time processing using proposed technique is possible [7]. The detection of the local disturbances, which occurs in the noise, gives a possibility of TBD algorithm application. It should be noted, that number of samples used for the analysis using the window approach is very low (e.g. 20 in presented examples). The chi-square and minimal chi-square preprocessing are very flexible and there are no limitations related to the distribution shape. The additive Gaussian noise is assumed mostly in this work, but other distributions are possible to compare.

The performance of the proposed technique will be considered in the further works. Optimization of the window size for high spatial resolution is also interesting research area.

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References


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